Optimal chirped probe pulse length for terahertz pulse measurement

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Abstract: A detailed analysis of the relationship between the duration of the chirped probe pulse and the bipolar terahertz (THz) pulse length in the spectral encoding technique is carried out. We prove that there is an optimal chirped probe pulse length (or an optimal chirp rate of the chirped probe pulse) matched to the input THz pulse length and derive a rigorous relationship between them. We find that only under this restricted condition the THz signal can be correctly retrieved.

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References and links

For past two decades THz pulse or T-ray detection techniques have been developed due to the requirements to numerous applications of THz science including material characterization, biomedical imaging, and tomography. One of the typical T-ray detection techniques is electro-optic sampling (EOS), a scanning technique. Through EOS, the electric field profile including phase and amplitude of a THz pulse can be measured. The main benefit of this technique is of its high temporal resolution depending on the short probe pulse length. However, the measured THz profiles represent averaged waveforms and a long time is required to obtain each profile. In addition, EOS is not suitable to be used in some cases such as the T-rays with low repetition rate or with strong shot-to-shot fluctuations and experiments.
that may have low duty cycles. To solve these problems three different single-shot detection techniques were developed. Among these techniques are the popular and practical spectral encoding and the cross-correlation techniques which normally are used for different cases. Compared to each other, the spectral encoding technique is more convenient to use due to its simple optical arrangement, capability of measuring the THz signal in real-time and providing THz spatiotemporal imaging with its disadvantage of limited temporal resolution. The cross-correlation technique has higher resolution which only depends on the duration of the short probe pulse while it has some disadvantages such as its more complicated optical arrangement and its absence of the ability of spatiotemporal imaging. To enhance the temporal resolution of the spectral encoding technique, an interferometric retrieval algorithm was proposed and was applied in experiments recently. This technique can provide transform-limited temporal resolution which is mainly limited by the spectral bandwidth of the optical probe pulse, regardless of its chirp. However, with this technique a complicated matrix inversion equation needs to be resolved numerically to retrieve the THz waveform. Furthermore, this algorithm needs to be improved due to the poor signal-to-noise ratio and the dominant presence of algorithm artifacts such as the fast oscillations when it was applied in some experiments.

The spectral encoding technique not only has been applied to many experiments but also has been analyzed theoretically by some authors after it was proposed. In their detailed analysis the temporal resolution of the detection system was obtained as where is the original probe pulse length and is the chirped probe pulse length. It was proposed that the THz signal could be retrieved without distortion if the pulse length of the detected THz field satisfies the condition . Though their theory can explain some experimental results, we found that this condition is a relatively inaccurate description. According to Sun’s analysis, to reduce the distortion, must be reduced so that the temporal resolution could be improved while the smallest should be larger than the THz duration. Hence a compromise between the temporal resolution and the chirp rate is desirable. In other words, there would be an optimal chirp rate or an optimal chirped probe pulse length that can be applied to measure the THz signal without distortion if the THz pulse length is given. In this paper we present a detailed analysis and a rigorous deduction of the relationship between the chirped probe pulse length and the THz pulse length . We prove that there is an optimal chirped probe pulse length that matches with the input THz pulse length and only under this restricted condition the THz signal can be retrieved properly.

For the sake of argument, we use the same definitions of the electric field of the original probe pulse, the chirped probe pulse, and the THz pulse as described in reference in following deduction. The electric field component of the original probe pulse with a central frequency and an envelope Gaussian function can be written as

Here is the pulse length, which is related to the laser spectral bandwidth through . After stretched by a grating pair or a dispersive glass, the electric field component of the probe pulse can be written as where is the pulse length after chirping, and is the chirp rate which is approximated as the laser bandwidth divided by the chirped laser pulse length, i.e. . Assuming a bipolar THz waveform as here we define a characteristic time which is the interval between the maximum and the minimum and define as the THz pulse length for simplicity.

When the chirped probe pulse and a THz pulse co-propagate in an electro-optic crystal, the chirped pulse is modulated by the pulsed THz field through the Pockels effect. Considering
the case that the polarizer and the analyzer are crossed to each other, the electric field component of the chirped probe beam modulated by the THz field can be written as:

\[ E_m(t) = E_c(t)(1 + kE_{THz}(t - \tau)) = E_c(t) + kE_{THz}(t - \tau)E_c(t) \]

with \( E_{THz}(t) \) the electric field of the THz waveform, \( \tau \) is the relative time delay between the probe pulse and the THz pulse, and \( k \) is the modulation constant. Here we assume \( \tau = 0 \) for simplicity due to the fact that the probe pulse can be synchronized with the THz pulse by adjusting the time delay in principle. Thus we have

\[ E_m(t) = E_c(t) + kE_{THz}(t)E_c(t) = E_c(t) + E_\perp(t) \]

The spectral modulation is spatially separated on the CCD of the spectrometer. Assuming that the measured signal on a CCD pixel corresponds to the optical frequency \( \omega \), and considering 10 proportional to the time through measurements of the spectral modulation \( I \) can be expressed as the convolution of the spectral function and the square of the spectral resolution function of the spectrometer is

\[ I(\omega) = \int_{-\infty}^{\infty} \delta(\omega_1 - \omega) |E_c(\omega)|^2 d\omega = |E_c(\omega)|^2 \]

and

\[ I_m(\omega) = g(\omega_1 - \omega) |E_m(\omega)|^2 \]

Here \( E_c(\omega) \) and \( E_m(\omega) \) are the Fourier transform of \( E_c(t) \) and \( E_m(t) \), respectively. If the spectral resolution is large enough, the spectral function of the spectrometer can be expressed as a \( \delta \) function. Thus we have

\[ I_c \approx \delta(\omega_1 - \omega) |E_c(\omega)|^2 = \int_{-\infty}^{\infty} \delta(\omega_1 - \omega) |E_c(\omega)|^2 d\omega = |E_c(\omega)|^2 \]

and

\[ I_m \approx g(\omega_1 - \omega)|E_m(\omega)|^2 = \int_{-\infty}^{\infty} \delta(\omega_1 - \omega)|E_m(\omega)|^2 d\omega = |E_m(\omega)|^2 \]

The difference between \( I_m \) and \( I_c \) can be expressed as

\[ \Delta I \approx I_m - I_c = |E_m(\omega_1)|^2 - |E_c(\omega_1)|^2 \]

Considering \( E_{\alpha_1}(\omega_1) = \hat{E}_c(\omega_1) + \hat{E}_s(\omega_1) \), we have

\[ \Delta I \approx |E_c(\omega_1) + E_s(\omega_1)|^2 - |E_c(\omega_1)|^2 = E_c(\omega_1) \cdot E\perp_s(\omega_1) + E\perp_c(\omega_1) \cdot E_s(\omega_1) + |E_s(\omega_1)|^2 \]

where \( E\perp_c(\omega_1) \) and \( E\perp_s(\omega_1) \) are the conjugate of \( E_c(\omega_1) \) and \( E_s(\omega_1) \), respectively.

For a small amplitude of \( E_{THz}(t), k << 1, |E_c(\omega_1)|^2 \) is negligible and thus can be ignored. Then we have

\[ \Delta I \approx E_c(\omega_1) \cdot E\perp_s(\omega_1) + E\perp_c(\omega_1) \cdot E_s(\omega_1) \]

Defining a function \( S(\omega_1) \) which is related to the THz signal as follows:

\[ S(\omega_1) = E_c(\omega_1) \cdot E\perp_s(\omega_1) + E\perp_c(\omega_1) \cdot E_s(\omega_1) \]

(1)

For a linear chirp, the probe instantaneous frequency \( \omega_1 \) measured on the CCD is proportional to the time \( t' \). In other words, the THz field profile can be directly obtained through measurements of the spectral modulation \( S(\omega_1) \) using following coordinate transformation \( \omega = \alpha \omega_1 + 2\alpha t' \). Due to \( T_0 = 2/\Delta \omega_0 \) and \( 2\alpha \approx \Delta \omega_0/Tc \) as mentioned before, we have \( \alpha = T_0^{-1}T_c^{-1} \). Substituting \( E_0(t_0), E_c(t_0) \), and \( E_{THz}(t) \) which we have defined above in Eq. (1), and considering \( \omega \approx \omega_0 = 2\alpha t' \) and \( \alpha = T_0^{-1}T_c^{-1} \), and introducing two dimensionless pulse lengths \( m = T/T_0 \) and \( n = T_0/T_c \), one obtains

\[ S(t') = kT_0^{-1}n^{-1}(-t'T^{-1}) \exp((-t'^2T^{-2}) \cdot \chi) \]

(2)

with

\[ \chi = 2\pi T_0^{-1}n^{-1}(m^2 + n^2)^{-1/2} \left[(m^2 + n^2)^2 + n^{-4} + m^{-4} \right]^{3/2} \left[(m^2 + n^2) + n^{-2} \right]^{1/4} \]

(3)
Given $T$ and $T_c$, $\gamma$ is a constant which does not affect the profile of the reconstructed THz field. Due to the fact that it is in the exponent which is related to $t'$, $\chi$ is the main factor which affects the profile of the reconstructed THz field. Though the function $f(t'/T_0)$ has a plateau around the center of the time window and it oscillates at both sides of the time window hence introducing distortion to some extent at the two ends of the retrieved THz signal. An example of this function as well as the original THz field when $m = 6$ are shown in Fig. 1. To eliminate the possible distortion generated from function $f(t'/T_0)$, we define a normalized THz signal $S_n(t')$ to $f(t'/T_0)$, obtaining

$$f(t'/T_0) = n^{-1} \cos \left( \frac{1}{2} \arctg \left( \frac{n^3}{m^2 n^2 + m^2 + n^2} \right) - \frac{n^3 (2m^2 + n^2)}{(1+n^2) \left[ (m^2 + n^2)^2 + m^2 n^2 \right]} \left( \frac{t'}{T_0} \right) \right)$$

$$-(m^2 + n^2) \sin \left( \frac{1}{2} \arctg \left( \frac{n^3}{m^2 n^2 + m^2 + n^2} \right) - \frac{n^3 (2m^2 + n^2)}{(1+n^2) \left[ (m^2 + n^2)^2 + m^2 n^2 \right]} \left( \frac{t'}{T_0} \right) \right)$$

(4)

$$\chi = \frac{m^2}{1+n^2} + \frac{m^4 n^2 + m^6}{(m^2 + n^2)^2 + m^4 n^2}$$

(5)

$$S_n(t') = \frac{S(t')}{f(t'/T_0)} = k \gamma \cdot (-t'T^{-1}) \exp \left[ (\frac{2}{t'}^3 \cdot \chi) \right]$$

(6)

Fig. 1. An example of the oscillation function $f(t'/T_0)$ (solid line) as well as an original THz field (dashed line) when $m = 6$. 

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One can see clearly from Eq. (6) that the function \( S_n(t') \approx E_{\text{THz}}(t') \) if \( \chi = 1 \). In other words, the real THz profile can be retrieved if \( \chi = 1 \) while severe distortion would be introduced if \( \chi > 1 \) or \( \chi < 1 \). When \( m > 1.4 \) (this condition can be easily be fulfilled due to the fact that \( T >> T_0 \)) then the only reasonable solution of equation \( \chi = 1 \) follows as

\[
n = \left\{ \frac{1}{3} \left[ 2(6m^6 + 4m^4 - 8m^2 + 1)^{1/2} \cos \left( \frac{\beta}{3} \right) - (m^2 - 1) \right] \right\}^{1/2}
\]

with \( \beta = \arccos \left[ -\frac{3}{54} \frac{18m^8 + 61m^6 + 6m^4 + 24m^2 - 2}{(6m^6 + 4m^4 - 8m^2 + 1)^{3/2}} \right] \). Here \( n \) is the dimensionless optimal probe pulse length. Thus we obtain the optimal duration \( T_{\text{o}} = nT_0 \) and the optimal chirp rate \( 2\alpha_{\text{o}} = 2T_0^{-1}T_{\text{o}}^{-1} = 2T_0^{-2}n^{-1} \). This means that it is possible to retrieve the THz signal without distortion if one uses a chirped probe pulse with the optimal duration \( T_{\text{o}} \) matched to an input THz pulse length \( T = mT_0 \) according to Eq. (7). The relationship between \( m \) and \( n \) according to Eq. (7) is shown in Fig. 2. One can see that the dimensionless optimal chirped probe pulse length \( n \) increases nonlinearly with the input dimensionless THz pulse length \( m \).

![Fig. 2. Relationship (solid line) between the dimensionless optimal chirped pulse length \( n \) and the input dimensionless THz pulse length \( m \). The restricted condition \( T_{\text{o}} \leq T/T_0 \) or \( T \geq (T_0T_{\text{o}})^{1/2} \) described by Sun’s theory is also shown as the area under the curve \( n = m^2 \) (or \( T_{\text{o}} = T/T_0 \), dashed line) and the curve itself compared with the optimal \( T_{\text{o}} \) curve.](image)

By comparing between the condition described by Sun \(^9\) and Fletcher \(^10\) and the relationship of Eq. (7), one can find that their description is different from ours. In their theory,
Fig. 3. (Color on line) Retrieved THz waveforms vs the original THz waveforms (black dotted lines) with $T_0=25\text{fs}$ in (a) and $T_0=10\text{fs}$ in (b). The red solid lines are the retrieved THz waveforms calculated according equation (6) under the conditions of input THz pulse length $T = 0.5\text{ps}$ ($m = 20$ when $T_0=25\text{fs}$ and $m = 50$ when $T_0=10\text{fs}$) and their corresponding optimal chirped pulse lengths $T_{co} = 2.6437\text{ps}$ when $T_0=25\text{fs}$, $T_{co} = 4.1943\text{ps}$ when $T_0=10\text{fs}$, respectively, which are calculated from equation (7). The retrieved THz fields calculated according to the equation (6) with $T_c = T_{co}/2$, $T_c = T_{co}/4$, $T_c = 2T_{co}$, and $T_c = 4T_{co}$ are also shown in each figure.
the condition to retrieve the original THz field without distortion was described as $T \geq (T_0 T_c)^{1/2}$ or $T \leq T_0^2 / T_c$ (i.e. $n \leq m^2$ if we introduce the dimensionless pulse length). The curve of $n=m^2$ (marked “$T=T_0^2 / T_c$”) and the curve of the optimal $T_c$ (marked “optimal $T_c$”) obtained from Eq. (7) are shown in Fig. 2. One can find that the condition “$T \leq T_0^2 / T_c$” is inaccurate as this condition includes the whole area under the curve $n=m^2$ and the curve itself, while our analysis indicates that only those chirped probe pulses with duration $T_c$ satisfy the curve obtained from equation (7) can be applied to reconstruct the original THz signal.

To verify the validity of our theory, we calculated a representative example with the original THz pulse length $T=0.5$ps. We assumed in our calculation that the Fourier-limited pulse length $T_0=25$fs and $T_0=10$fs. According to Eq. (7) the optimal chirped probe pulse lengths $T_c$ matched to above $T$ can be calculated to be as 2.6437ps when $T_0=25$fs and 4.1943ps, when $T_0=10$fs, respectively. With these $T$ and their $T_c$, we calculated the THz fields according to Eq. (6). The results are shown in Fig. 3 in which $T_0=25$fs in (a) and $T_0=10$fs in (b). In Fig. 3, the dotted black lines represent the original THz fields, while the solid red lines are the retrieved THz fields calculated with the optimal probe pulse length $T_c$. One can find that the retrieved THz fields with the $T_c$ match the original ones very well.

To see the behavior of the retrieved THz field due to the discrepancy $\Delta T_c=T_c-T_c$, we calculated four cases with $T_c=T_c/2$, $T_c=T_c/4$, $T_c=2T_c$, and $T_c=4T_c$ which fulfill the condition $T \geq (T_0 T_c)^{1/2}$ of Sun’s theory. The results are also shown in Fig. 3. One can clearly see that all these cases introduce distortions and the larger the difference of $T_c$ to $T_c$ is, the more severe the distortion of the retrieved THz field will be. In detail, if $T_c<T_c$, the retrieved THz waveform will be compressed while its spectrum would be broaden compared to the input original THz field. Whereas, if $T_c>T_c$, the retrieved THz waveform will be broaden while its spectrum would be compressed.

In addition, one can find that the larger the $T_0$ is, the greater the difference of the retrieved THz waveform to the same discrepancy $\Delta T_c$ will be if one compares Fig. 3(a) with Fig. 3(b). In Fig. 3(b), the distortion of the retrieved THz fields is very small even if the discrepancy $\Delta T_c$ is relatively large ($\Delta T_c=2.097155$ps in case of $T_c=T_c/2$ and $\Delta T_c=4.1943$ps in the case of $T_c=2T_c$), while in Fig. 3(a) the distortion of the retrieved THz fields is more severe than that shown in Fig. 3(b) even if their $\Delta T_c$ ($\Delta T_c=1.32185$ps in the case of $T_c=T_c/2$ and $\Delta T_c=2.6437$ps in the case of $T_c=2T_c$) is smaller than that of Fig. 3(b). In other words, if an input THz pulse is measured using different probe beams, the shorter the original probe pulse length (or the broader the spectrum of the original probe pulse) is, with the same discrepancy to their optimal chirped pulse length, the smaller the distortion of the retrieved THz signal will be.

We also simulated the whole retrieving process of the THz waveforms under the condition of $k<<1$ (for example $k=0.02$). We find that the simulation results are the same to the ones shown in Fig. 3. Our simulations confirm our deduction process and prove the validity of Eq. (7). The detail of these simulation results will be published in the future. It should be noted that our analysis is based on a bipolar THz field. For other THz pulses such as with multiple cycles, we prove that there still is an optimal chirped probe pulse duration $T_c$ by means of our simulation, while the characteristic time of the THz field should to be redefined and the expression of Eq. (7) needs to be modified. The detailed theoretical deduction will be performed in the future work.

In summary, we conduct a rigorous deduction and a detailed analysis of the relationship between the chirped pulse length $T_c$ and the input bipolar THz pulse length $T$ in the spectral-
encoding technique. We prove that there is an optimal chirped pulse duration $T_{co}$ or an optimal chirp rate matched to the input THz pulse length $T$. We derive a relationship between the dimensionless optimal chirped pulse length $n$ and the input dimensionless THz pulse length $m$. With this relationship the optimal duration and the optimal chirp rate of the chirped probe pulse can be calculated according to $T_{co} = nT_0$ and $2\alpha = 2T_0^{-1}\alpha_0^{-1} = 2T_0^{-2}n^{-1}$ respectively. We find that only under this restricted condition the measured THz signal can be retrieved without distortion.

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